Cruise Control

MEM 355 Performance Enhancement of Dynamical Systems

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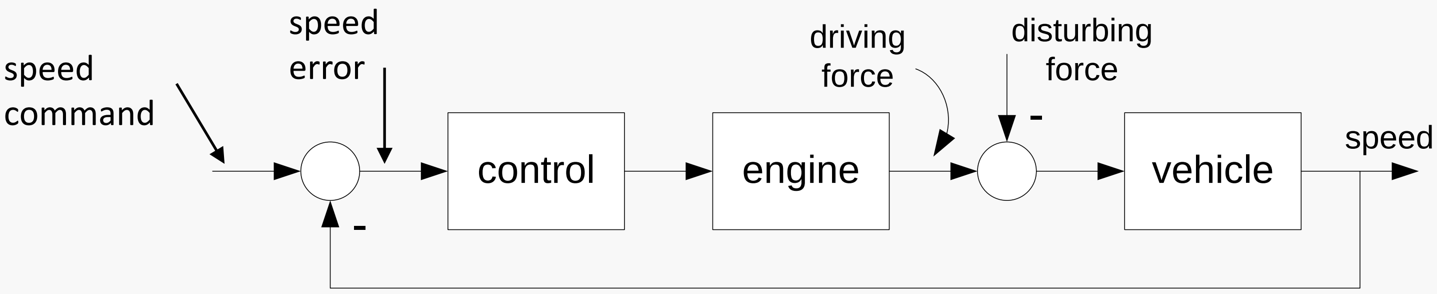
Cruise Control 1

$$m\dot{v} = F - mg\sin\theta(t) - cv$$
$$\dot{v} + 0.02v = u - 9.8\theta$$

$v\,[m/s]$ speed (10 m/s=36 km/h=22 miles/hr)

u normalized throttle $0 \le u \le 1$

$\theta\,[rad]$ roadway slope



Criteria

- What characteristics do we expect in a good cruise control system?
  - Ultimate tracking error in response to speed command
  - Ultimate tracking error in response to disturbance
  - Transient response time
  - Transient overshoot, undershoot
  - Robustness – tolerance to unmodeled dynamics or parameter variation

Cruise Control: derive error response from differential equations

Assumptions: 'proportional' plus 'integral' control, ignore engine dynamics

$$u(t) = k_p\left(\bar{v}(t) - v(t)\right) + k_i \int_0^t \left(\bar{v}(\tau) - v(\tau)\right) d\tau$$

Define: $e(t) = \bar{v}(t) - v(t)$

$\dot{v} + 0.02v = u - 9.8\theta \Rightarrow \ddot{e} + 0.02\dot{e} = -\dot{u} + 9.8\dot{\theta} + \ddot{\bar{v}} + 0.02\dot{\bar{v}}$

$$u(t) = k_p\left(\bar{v} - v(t)\right) + k_i \int_0^t \left(\bar{v} - v(\tau)\right) d\tau \Rightarrow \dot{u} = k_p\dot{e} + k_i e$$

$$\Downarrow$$

$$\boxed{\ddot{e} + \left(0.02 + k_p\right)\dot{e} + k_i e = 9.8\dot{\theta} + \dot{\bar{v}} + 0.02\bar{v}}$$

$$E(s) = \frac{(s + 0.02)}{s^2 + \left(0.02 + k_p\right)s + k_i}\bar{V}(s) + \frac{9.8s}{s^2 + \left(0.02 + k_p\right)s + k_i}\Theta(s)$$

e speed error

2 Inputs:

$\bar{v}$ speed command

θ road disturbance

Error response to command

Error response to disturbance

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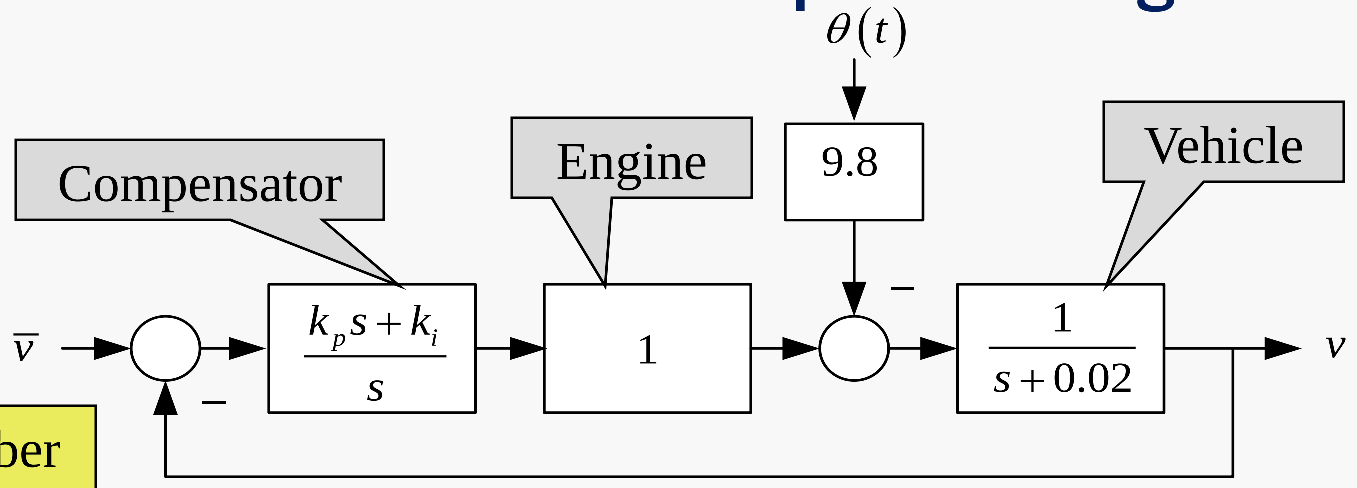
Cruise Control: derive error response using block diagrams



Remember the formula

$$\bar{v} \to e : G_{\bar{v}} = \frac{1}{1 + \frac{k_p s + k_i}{s} \frac{1}{s+0.02}} \Rightarrow \boxed{G_{\bar{v}} = \frac{(s+0.02)}{s^2 + (0.02 + k_p)s + k_i}}$$

Or do the algebra

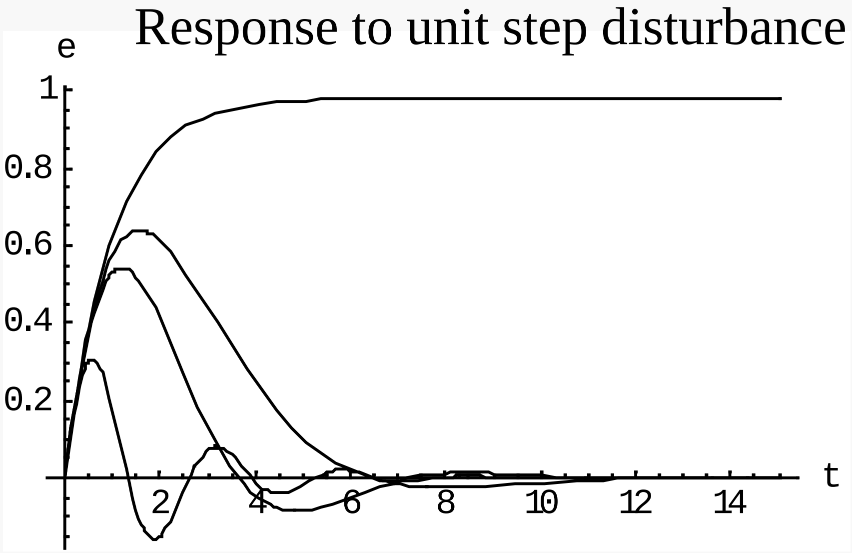
$$\theta \to e : E = -\frac{1}{s+0.02}(-9.8\Theta) - \frac{k_p s + k_i}{s} E \Rightarrow$$

$$\left[1 + \frac{k_p s + k_i}{s}\right] E = \frac{9.8}{s+0.02}\Theta \Rightarrow \boxed{G_\theta = \frac{9.8s}{s^2 + (0.02 + k_p)s + k_i}}$$

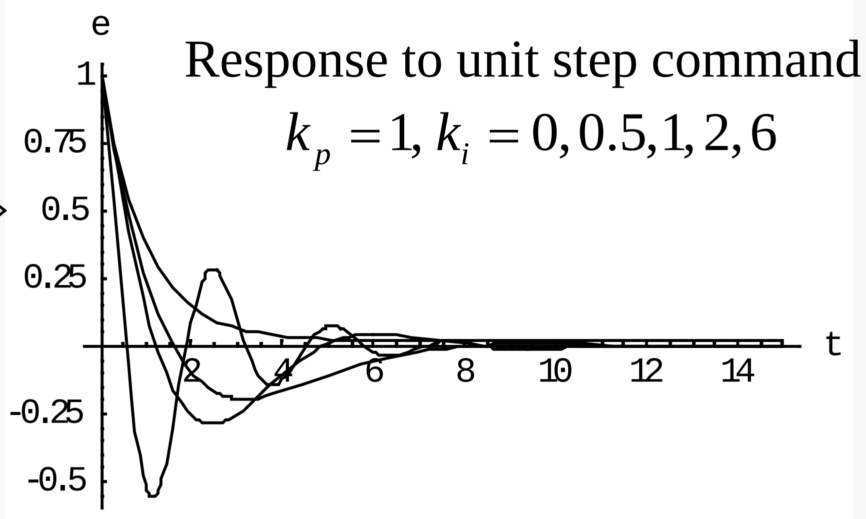
Closed loop transfer functions with two design parameters

Cruise Control 4

$$E(s)=\frac{9.8s}{s^2+(0.02+k_p)s+k_i}\Theta(s),\quad \Theta(s)=\frac{1}{s}$$



$$E(s)=\frac{(s+0.02)}{s^2+(0.02+k_p)s+k_i}\bar{V}(s),\quad \bar{V}(s)=\frac{1}{s}$$



Observations

- Both transfer functions have same denominator (same poles), but different numerators (different zeros)
- When $k_i = 0$ (proportional control) the ultimate error is not zero, in fact the ultimate error in response to command is very small, but to disturbance is large.
- For stability we can look at either transfer function, but for performance we need to consider both.
- To evaluate k_p, k_i it is helpful to make the association

$$s^2 + (0.02 + k_p)s + k_i \Leftrightarrow s^2 + 2\rho\omega_0 s + \omega_0^2$$

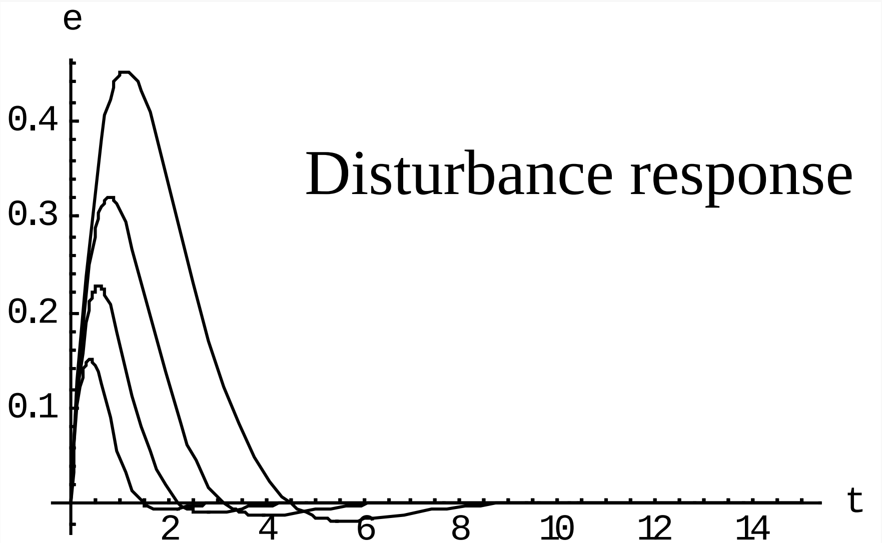
Refine the Control

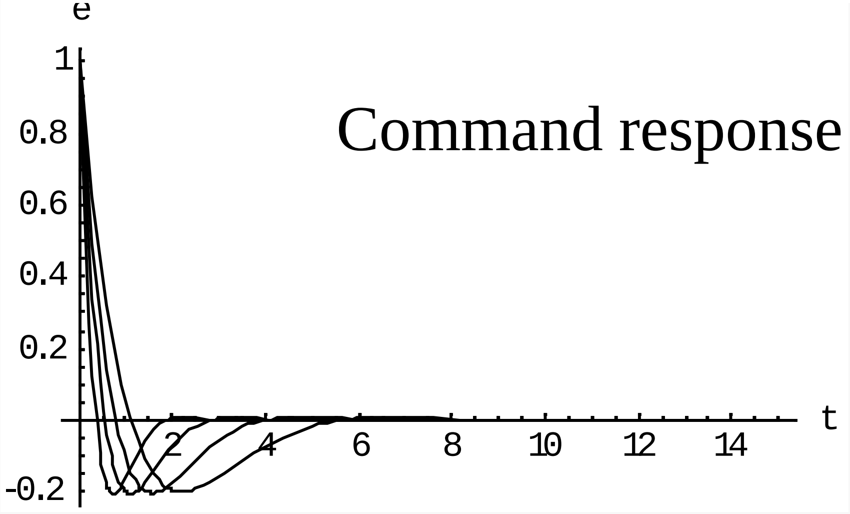
Notice that we can specify, ρ, ω_0 and choose

$$k_i = \omega_0^2, k_p = 2\rho\omega_0 - 0.02$$

Let us fix $\rho = 0.707$ and look at

$$\omega_0 = 1, \sqrt{2}, 2, 3$$





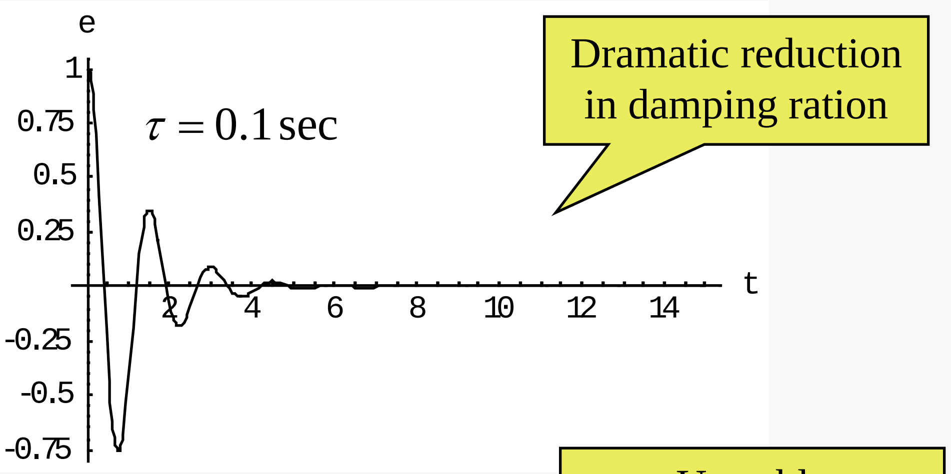
Effect of Engine Dynamics

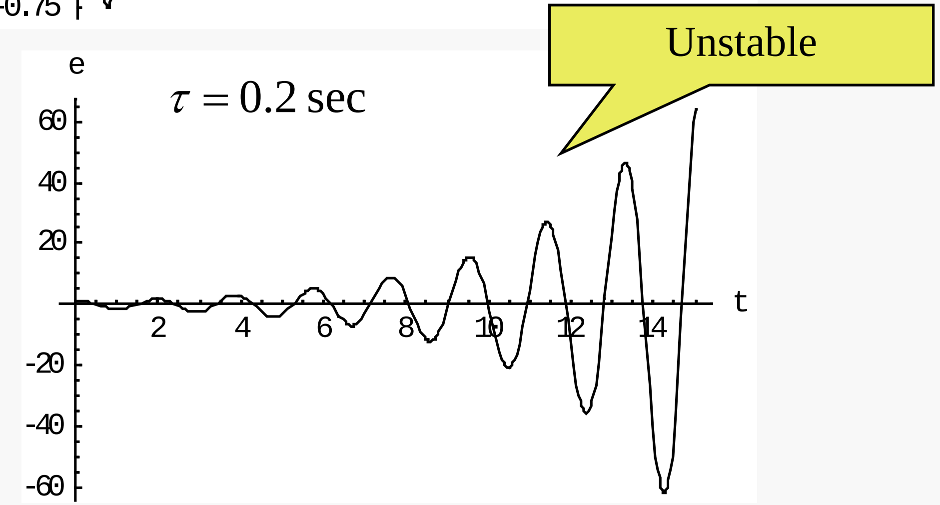
Error response to command step using the <u>fastest controller</u>.

$$\rho = 0.707, \omega_0 = 3$$

Engine time constant 0.1 sec, and 0.2 sec.

$$G_{eng} = \left(\frac{1}{\tau s + 1}\right)^2, \tau = 0.1, 0.2$$



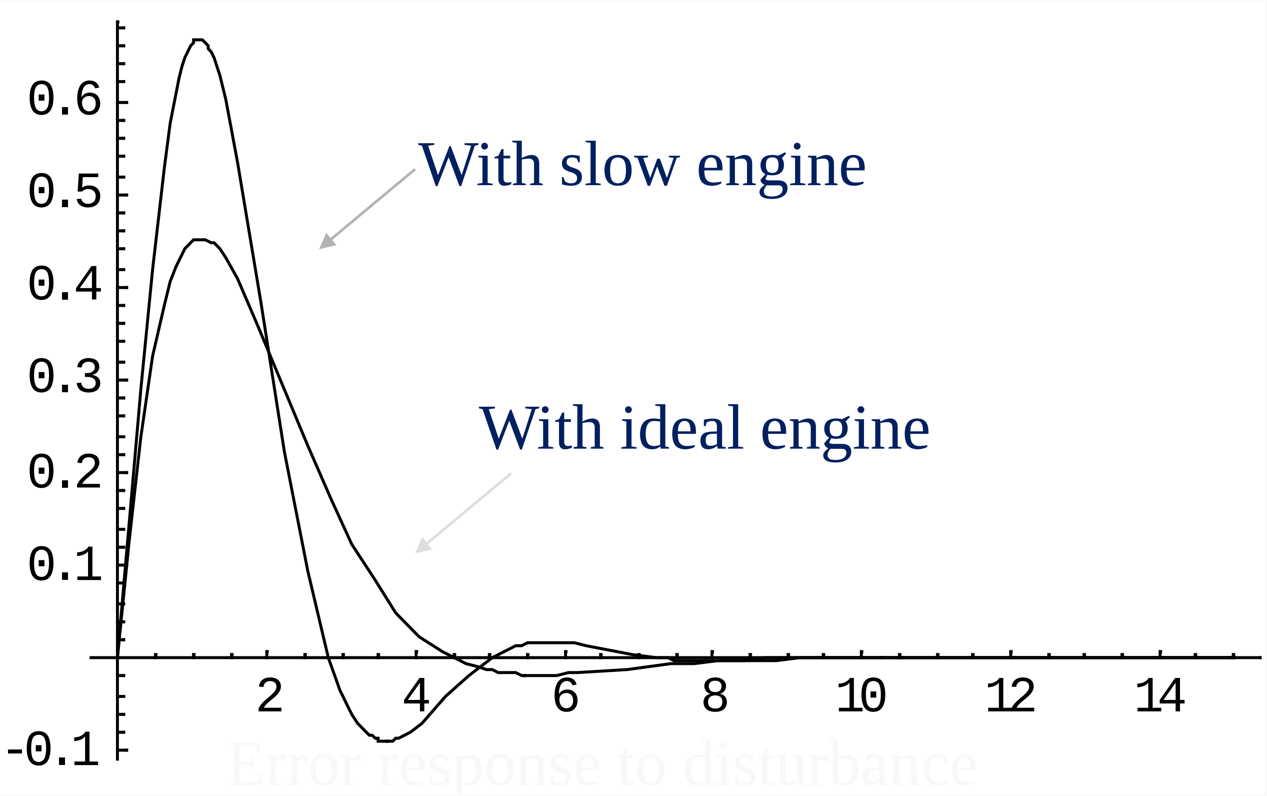


Effect of Engine Dynamics ~ 2

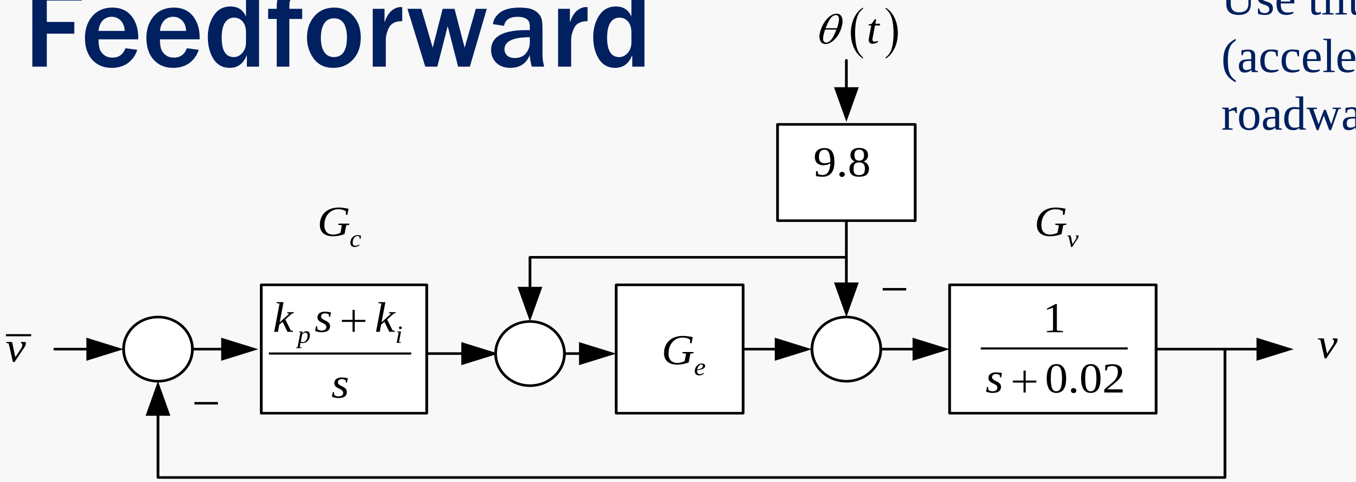
- Suppose we use the <u>slowest controller</u>

$$\rho = 0.707, \omega_0 = 1$$

- Here we see how the response degrades when slow engine is included, at least it is still stable
- Pushing for high performance often leads to non-robust design.

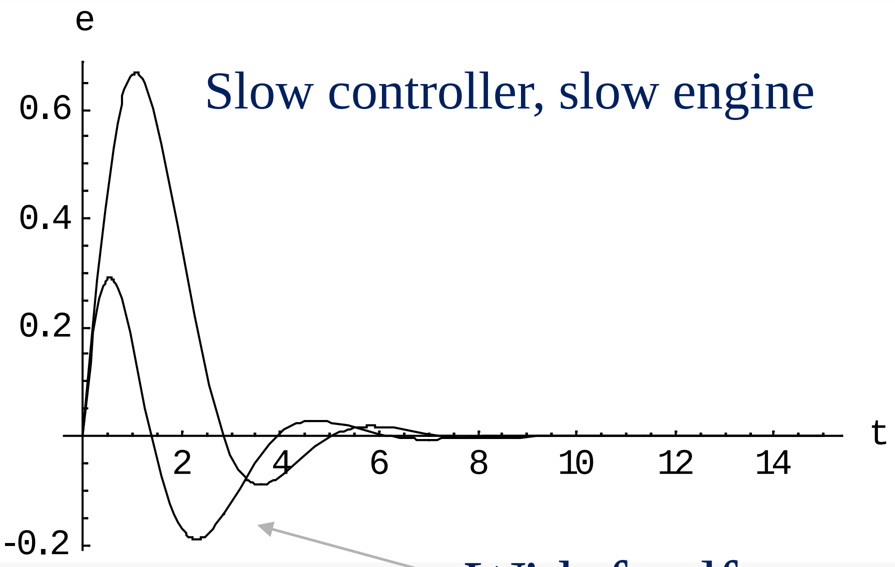


Add Feedforward



Use tilt sensor (accelerometers) to estimate roadway slope.

$$E = \frac{G_v\left(1 - G_e\right)}{1 + G_v G_e G_c} D$$



With feedforward

Summary

- Any one of the closed loop transfer functions can be used for stability analysis (all have same poles)
- Performance analysis usually requires considering two or more closed loop transfer functions.
- Ultimate error depends on controller type, e.g. PI controller resulted in zero error eventually, but P controller left some residual error – nontrivial in the case of disturbance.
- We can choose control parameters to shape transient response (locate closed loop poles) – in this special case we used our knowledge of 2^{nd} order system behavior.
- The system may be sensitive to model accuracy, including neglected dynamics – even to the point of instability.

Next Steps

- Controlling the ultimate error
- Evaluating closed loop system ‘stability robustness’ – the ability to remain stable when the model is uncertain
- Shaping the transient response: closed loop pole location via root locus
- Controller design to achieve robust performance
- Other (more direct) methods for shaping transient response